VOLUME

I

SIGMA NOTATION

VOL I

MATHEMATICAL FOUNDATIONS IN STATISTICS



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Written by Andrea Millán, Ph.D.

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Written by PhD. Andrea Millán

Table of Contents

Introduction to Sigma Notation 3

The Structure of Sigma Notation 4

Key Properties of Summation 5

Practical Examples 6

Acknowledgements 8

Introduction to Sigma Notation

Sigma notation (or summation notation), denoted by the Greek letter Σ, is a fundamental tool in mathematics, statistics, and other sciences. Its primary purpose is to express the sum of a series of numbers or terms in a concise and manageable way. Instead of writing a long sum like $x_1+x_2+x_3+x_4+x_5$, sigma notation allows it to be condensed into a compact expression such as $\sum_{i=1}^{5} x_i$.

This type of notation is particularly useful for **statistical analysis**, as it simplifies the representation of common calculations like the mean of a data set, where all values are summed and divided by the number of observations. It is also crucial in fields such as calculus, physics, and computer science, where it is necessary to work with series and optimize algorithms.

The Structure of Sigma Notation

The basic structure of sigma notation is composed of several key elements:

Summation Symbol (Σ): This is the heart of the notation, indicating that a sum must be performed. **Index of Summation (i)**: A variable that iterates through a range of integer values, defining the terms to be summed. Letters like i, j, or k are often used. **Limits of Summation**: These limits define the range of the index. **Lower Limit**: The value where the index begins ($i = 1$). **Upper Limit**: The value where the index ends (e.g., n or k). **Term Expression (f_i)**: This is the mathematical formula that defines each term in the sequence to be summed. It generally includes the index variable.

The general notation is presented as follows:

$$\sum_{i=j}^{k} f_i$$

In this expression, Σ is the summation symbol, i is the index, j is the lower limit, k is the upper limit, and f_i is the term expression.

Key Properties of Summation

Sigma notation possesses properties that make it a versatile tool for simplifying and manipulating complex mathematical expressions:

- **Linearity**: The sum of two series is equal to the sum of each series separately.
  - $$\sum_{i=j}^{k}(a_i + b_i) = \sum_{i=j}^{k} a_i + \sum_{i=j}^{k} b_i$$
- **Constants**: A constant multiplied by a variable can be factored out of the summation.
  - $$\sum_{i=j}^{k}(c \cdot a_i) = c \cdot \sum_{i=j}^{k} a_i$$
  - The sum of a constant c for n times is equal to $n \cdot c$.
  - $$\sum_{i=1}^{n} c = n \cdot c$$
- **Reordering of Terms**: The order in which the terms are summed does not alter the result.

Practical Examples

Example 1: Calculating a Simple Sum

Suppose you want to calculate the sum of the following series: $\sum_{i=2}^{4} x_i$ with the numbers $3, 0, 5, 9, 2, 7$.

The summation starts with the second term (x_2) and ends with the fourth term (x_4). The corresponding values are $0, 5, 9$.

Solution: $0 + 5 + 9 = 14$

Example 2: Sum of Squared Terms

For the notation $\sum_{i=2}^{n} x_i^2$ with the same data, the sum starts with the second term and each value is squared.

Solution: $0^2 + 5^2 + 9^2 + 7^2 = 0 + 25 + 81 + 49 = 155$

Note: The terms are squared up to the last one (n) in the data set.

Example 3: Application in Statistics

An important result in statistics is demonstrated by subtracting the mean ($\bar{x}$) from each element of a data set and then summing the results.

$$\sum_{i=1}^{n}(x_i - \bar{x}) = \sum_{i=1}^{n} x_i - \sum_{i=1}^{n} \bar{x}$$

(by the linearity property)

$$= \sum_{i=1}^{n} x_i - n\bar{x}$$

(by the constant property)

Since the definition of the mean is $\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$, then $\sum_{i=1}^{n} x_i = n\bar{x}$.

$$= n\bar{x} - n\bar{x} = 0$$

This demonstrates that the **sum of the deviations of the values from their mean is always zero.**

Acknowledgements

This eBook, titled **"Volume I: Sigma Notation"** was written by Andrea Millán, who holds a Ph.D. in health sciences and has over ten years of experience in statistical analysis for epidemiological studies. This material was developed in 2025 with the goal of offering a clear and practical introduction to sigma notation, aimed at students and professionals in statistics.

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