VOLUME

IV

PERMUTATIONS

VOL IV



MATHEMATICAL FOUNDATIONS IN STATISTICS

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Introduction to Permutations

Permutations are a fundamental concept in probability theory and combinatorics, essential for the rigorous analysis of discrete events. Unlike combinations, a permutation is an **ordered arrangement** of elements from a set.

For example, if you have the numbers 1, 2, and 3, the possible permutations are: (1, 2, 3), (1, 3, 2), (2, 1, 3), (2, 3, 1), (3, 1, 2), and (3, 2, 1). Each of these is a distinct combination. In statistics and probability, this is crucial for calculating the number of possible outcomes in events where the sequence or order of elements is relevant.

The Permutation Formula

The number of permutations of n elements taken k at a time is calculated using the following formula:

$$P(n, k) = \frac{n!}{(n-k)!}$$

* n: The total number of elements in the set. * k: The number of elements being selected or arranged at a time. * !: The factorial symbol, which we have already studied.

The formula uses the factorial to count all possible orderings and then divides by the factorial of the unselected elements. That is, for the first element, there are n options; for the second, $n-1$; and so on until the k-th element, for which there are $n-(k-1)$ options. This ensures that only the arrangements of the subset of size k are counted.

Relevant Properties

The Multiplication Rule A fundamental property that governs permutations is the **multiplication rule**. If one event has n_1 possible outcomes and a second event has n_2 possible outcomes, the total number of outcomes for both events is $n_1 \cdot n_2$. This rule is the logical basis for the permutation formula.

Complete Permutations $(k = n)$**:** If all elements of the set are taken, the number of permutations is simply the factorial of n.

$$P(n, n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

Practical Calculation Examples

Example: Permutations of 5 elements taken 2 at a time

How many ways are there to choose and order 2 people from a group of 5 to fill the positions of president and vice-president?

Here the order matters (being president is not the same as being vice-president). We use $n = 5$ and $k = 2$.

$$P(5, 2) = \frac{5!}{(5-2)!} = \frac{5!}{3!} = \frac{5 \cdot 4 \cdot 3!}{3!} = 5 \cdot 4 = 20$$

Calculation in R:

In R, there is no native `permutation()` function, but you can easily calculate it by applying the formula directly.

Example 1: Permutations of 5 elements taken 2 at a time In this case, we must apply the factorial notation to 5 and divide by the factorial of $(5 - 2)$.

```r
permutation_example1 <- factorial(5) / factorial(5 - 2)
print(permutation_example1)
```

```
## [1] 20
```

'permutation_example1' is the name we give to this operation. 'print(permutation_example1)' will return the value of the operation in the console.

Example 2: Permutations of 4 elements taken 4 at a time

For this example, all elements are selected, so $n = 4$ and $k = 4$. The formula simplifies to 4! since $(4 - 4)! = 0! = 1$.

$$P(4, 4) = \frac{4!}{(4-4)!} = \frac{4!}{0!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1} = 24$$

Calculation in R:

```r
# P(4, 4) = 4! / (4-4)!
permutation_example2 <- factorial(4) / factorial(4 - 4)
print(permutation_example2)
```

```
## [1] 24
```

the result is 24 arrangements of elements, in different order

R Calculation for Example 2:

- **Method 1. Applying the permutation formula**

```
permutation_example2 <- factorial(4) / factorial(4-4)
print(permutation_example2)
```

```
## [1] 24
```

- **Method 2. A simpler way to calculate it, using only the factorial**

```
factorial(4)
```

```
## [1] 24
```

Example 3: First 3 places in a race with 10 runners In a race with 10 runners, how many different ways can the first three places (gold, silver, and bronze) be filled? The order of arrival is crucial, so we use permutations. We have a total of $n = 10$ runners and we select $k = 3$ of them.

$$P(10, 3) = \frac{10!}{(10-3)!} = \frac{10!}{7!} = 10 \cdot 9 \cdot 8 = 720$$

R Calculation for Example 3:

```
permutation_example3 <- factorial(10) / factorial(10-3)
print(permutation_example3)
```

```
## [1] 720
```

Example 4: Creating a password with unique digits How many 4-digit passwords with different digits can be formed using the digits from 0 to 9? The word "different" is the key, as it means there is no repetition. Since the order matters for a password, we use permutations. We have a set of $n = 10$ digits and we select $k = 4$ without repetition.

$$P(10, 4) = \frac{10!}{(10-4)!} = \frac{10!}{6!} = 10 \cdot 9 \cdot 8 \cdot 7 = 5,040$$

R Calculation for Example 4:

```r
# P(10, 4) = 10! / (10-4)!
permutation_example4 <- factorial(10) / factorial(10 - 4)
print(permutation_example4)
```

```
## [1] 5040
```

Example 5: Creating a 4-digit password How many 4-digit passwords can be formed using the digits from 0 to 9, allowing repetition?

When repetition is allowed, the factorial logic no longer applies. In this case, for each of the 4 positions in the password, we have 10 options. The formula for permutations with repetition is n^k. In this case, 10^4.

$$10^4 = 10 \cdot 10 \cdot 10 \cdot 10 = 10000$$

R Calculation for Example 5:

```r
permutation_example5 <- 10^4

print(permutation_example5)
```

```
## [1] 10000
```

Acknowledgements

This eBook, titled **"Volume IV: Permutations"**, has been written by **PhD. Andrea Millán**, who holds a doctorate in health sciences and has over ten years of experience in statistical analysis for epidemiological studies. This material was developed in 2025 with the goal of offering a clear and practical introduction to sigma notation, aimed at students and professionals in statistics.

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