VOLUME

II

FACTORIAL NOTATION

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MATHEMATICAL FOUNDATIONS IN STATISTICS



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Table of Contents

Intoduction to Factorial Notation 3

The Components of Factorial Notation 4

Key Properties of the Factorial 6

Practical Calculation Examples 8
- Calculation with a Scientific Calculator 8
- Calculation in R 9

Acknowledgements 12

Intoduction to Factorial Notation

The **factorial** is a fundamental mathematical operation in disciplines such as combinatorial analysis, probability, and statistics. Its main purpose is to simplify the notation for a **successive product of positive integers**.

It is denoted by an exclamation mark (!) to the right of a non-negative integer. For example, instead of writing $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$, we can use the abbreviated notation 5!. This concept is crucial for counting the number of ways to order a set of objects, which is the basis of **permutations**.

The Components of Factorial Notation

To understand factorial notation, it's useful to break it down into its parts:

- **The Base Number (n):** This is the non-negative integer to which the operation is applied. For example, in 5!, the base number is 5. This number indicates where the multiplication starts.
- **Factorial Symbol (!):** This exclamation mark is an **operator** that transforms the base number. It indicates that we must multiply that number by all the positive integers that follow it in descending order, down to 1.
- **The Expansion:** The notation $n!$ is actually a compact way of representing a series of multiplications.

For any non-negative integer n, the factorial of n is defined as:

$$n! = n \cdot (n-1) \cdot (n-2) \cdots 3 \cdot 2 \cdot 1$$

Example: For 6!, the base number is 6. The operator ! tells us to expand the multiplication:

$$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$$

Key Properties of the Factorial

The factorial has very useful properties that allow us to understand its behavior and simplify calculations.

- **Recursive Relationship:** This property establishes that the factorial of a number can be expressed in terms of the factorial of a smaller number. It is one of the most important and is defined as:

$$n! = n \cdot (n-1)!$$

For example, $8! = 8 \cdot (7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1) = 8 \cdot 7!$.

This property is key to simplifying fractions with large factorials, as it allows us to **"decompose"** the larger factorial so that we can cancel out the common term. For example, $8! = 8 \cdot 7!$, which is very useful in divisions.

- **The Case of** $0!$**:** The definition that $0! = 1$ is not intuitive if we think of the factorial as a multiplication. However, this convention is crucial for maintaining **logical consistency** in mathematical formulas.

Logic: If we apply the recurrence property for $n = 1$, we get:

$$1! = 1 \cdot (1 - 1)! = 1 \cdot 0!$$

We know that 1! is simply 1. For the equality to hold true, the value of 0! must be 1. This definition also makes sense when considering arrangements: there is only one way to arrange zero objects (to simply do nothing).

- **Rapid Growth:** The value of $n!$ increases *extremely fast* as n grows. This means that even for relatively small values of n, the result is a very large number. For example, 13! is more than 6 billion.

Practical Calculation Examples

Factorial calculation can be performed manually for small numbers, with a scientific calculator for intermediate values, or with programming software like R for more advanced calculations.

Calculation with a Scientific Calculator

Most scientific calculators have a dedicated function for the factorial, usually labeled as $x!$ or $n!$.

- **Example:** To calculate 7! on a calculator:

1. Enter the number 7.
2. Locate and press the ! key (often found in the probability functions menu or above the division key).
3. The result will be 5040.

Calculation in R

R, being a programming environment and statistical software, includes a function to calculate factorials natively.

The main function we will use is factorial().

- **Example 1: Calculation of 4!**

R has a function called factorial() that takes a number and returns its factorial.This is equivalent to the manual operation $4 \cdot 3 \cdot 2 \cdot 1$.

```
factorial(4)
```

```
## [1] 24
```

- **Example 2: Calculation of 7!**

Here we see the efficiency of R. Although the number is larger, the function calculates it instantly, without the need for manual multiplication.

```r
factorial(7)
```

```
## [1] 5040
```

- **Example 3: Simplification of Fractions with Factorials** (8!/5!)

In practice, $8! = 8 \cdot 7 \cdot 6 \cdot 5!$, which allows us to simplify the fraction by canceling the 5! in the numerator and the denominator. The result is simply $8 \cdot 7 \cdot 6$.

- **Method 1:** Using the factorial() function for both numbersR calculates the full fraction.

```r
factorial(8) / factorial(5)
```

```
## [1] 336
```

- **Method 2: Using the simplification logic**

This is the simplified and more efficient version.

$$\frac{8 \cdot 7 \cdot 6 \cdot 5!}{5!} = 8 \cdot 7 \cdot 6 = 336$$

Acknowledgements

This eBook, titled **"Volume II: Factorial Notation"** was written by PhD. Andrea Millán, who holds a doctorate in health sciences and has over ten years of experience in statistical analysis in epidemiological studies. This material was prepared in the year 2025 with the objective of offering a clear and practical introduction to sigma notation, aimed at students and professionals of statistics.

We deeply appreciate your interest in this content and hope that it has been of great use. For more information about our eBooks, online courses, services, and other resources, please visit us on our website **biomedical-data.com**.



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