VOLUME

III

# COMBINATORICS

## VOL III

MATHEMATICAL FOUNDATIONS IN STATISTICS




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# Introduction to Combinations

Combinations are a way of counting the number of ways to select or group elements from a set, where **the order of the elements does not matter**. Unlike permutations, where an arrangement like (A, B) is different from (B, A), in combinations (A, B) and (B, A) are considered the same group.

This concept is essential in situations where only the composition of a group is important, not the sequence in which its members were chosen. For example, when selecting a team of people, choosing a set of ingredients for a recipe, or playing the lottery.

# The Combination Formula

The number of combinations of $n$ elements taken $k$ at a time is calculated using the following formula:

$$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

- $n$: The total number of elements in the set.

- $k$: The number of elements being selected at a time.
- !: The factorial symbol.
- $\binom{n}{k}$: This notation, known as the "binomial coefficient," is a common way to represent combinations.

The combination formula is a modification of the permutation formula. The additional $k!$ term in the denominator adjusts the result to remove the repetitions that would be generated by order, since order is not relevant in combinations.

# Key Properties of Combinations

Combinations have important properties that simplify calculations and are crucial in statistics and probability.

- **Symmetry:** One of the most useful properties is that the number of ways to choose $k$ elements from a set of $n$ is equal to the number of ways to choose the $n-k$ elements that are left out of the group.

$$\binom{n}{k} = \binom{n}{n-k}$$

If you choose 3 people from a group of 10 for a committee, it is the same as choosing the 7 people who will not be on the committee. The number of ways to do both is identical. This can simplify calculations if $k$ is a large number.

- **Pascal's Identity:** This property establishes a recursive relationship between binomial coefficients and is the basis of the famous Pascal's Triangle.

$$\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$$

To understand this, imagine you have a group of $n$ people and want to form a committee of $k$. You can divide the task into two cases: 1) if one specific person is on the committee, you need to choose $k-1$ more from the remaining $n-1$; or 2) if that person is not on the committee, you have to choose all $k$ members from the remaining $n-1$ people. The sum of both cases gives you the total.

- **Sum Identity:** The sum of all binomial coefficients for a fixed $n$ is equal to $2^n$.

$$\sum_{k=0}^{n} \binom{n}{k} = \binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{n} = 2^n$$

This represents the total number of subsets that can be formed from a set of $n$ elements. Each element can either be included or not included in a subset (2 options). If you have $n$ elements, the total number of subsets is $2^n$.

# Practical Calculation Examples

Unlike permutations, R does have a native function for calculating combinations, which greatly simplifies the process.

- **Example 1: Combinations of 5 elements taken 2 at a time**

How many different ways can a committee of 2 people be chosen from a group of 5? In this case, the order in which the people are chosen does not matter (it would be the same committee). Therefore, we use combinations. We use $n = 5$ and $k = 2$.

$$C(5, 2) = \frac{5!}{2!(5-2)!} = 10$$

**Calculation in R:**

- **Method 1: Apply the formula directly**

```
combination_example1 <- factorial(5) / (factorial(2) * factorial(5-2))


print(combination_example1)
```

```
## [1] 10
```

- **Method 2: Use R's native function choose()**

```
choose(n = 5, k = 2)
```

```
## [1] 10
```

---

- **Example 2: Use R's native function choose()**

In how many ways can you choose 5 cards from a standard deck of 52 cards?The order in which you receive the cards does not affect the hand you have, only the set of cards. We use $n = 52$ and $k = 5$.

$$C(52, 5) = \frac{52!}{5!(52-5)!} = \frac{52!}{5! \cdot 47!} = \frac{311875200}{120} = 2598960$$

**Calculation in R:**

- **Use the native function choose()**

```r
choose(n = 52, k = 5)
```

```
## [1] 2598960
```

---

- **Example 3: Forming a project team**

In a class of 30 students, in how many ways can a team of 4 people be formed for a project?We use $n = 30$ and $k = 4$.

$$C(30, 4) = \frac{30!}{4!(30-4)!} = \frac{30!}{4! \cdot 26!} = 27405$$

**Calculation in R:**

- **Use the native function choose()**

```r
choose(n = 30, k = 4)
```

```
## [1] 27405
```

# Acknowledgements

This eBook, titled **"Volume III: Combinatorics"**, has been written by **PhD. Andrea Millán**, who holds a doctorate in health sciences and has over ten years of experience in statistical analysis for epidemiological studies. This material was developed in 2025 with the goal of offering a clear and practical introduction to sigma notation, aimed at students and professionals in statistics.

We deeply appreciate your interest in this content and hope it has been of great use. For more information about our eBooks, online courses, services, and other resources, please visit us on our website **biomedical-data.com**.



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